Math 245

Spring 2022

Lecture 13

Solve $\quad x^3 - 9x^2 > 5x - 45$

$x^2 - 5 =$
$x^2 - (\sqrt{5})^2$
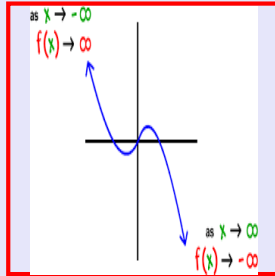
RHS=0 $\quad x^3 - 9x^2 - 5x + 45 > 0$

Factor LHS $\quad x^2(x-9) - 5(x-9) > 0$

$\qquad (x-9)(x^2-5) > 0$

$\qquad (x-9)(x+\sqrt{5})(x-\sqrt{5}) > 0$

Set each
Factor =0, Solve

$x-9=0$	$x+\sqrt{5}=0$	$x-\sqrt{5}=0$
$x=9$	$x=-\sqrt{5}$	$x=\sqrt{5}$

Sign chart

x	$-\infty$	$-\sqrt{5}$	$\sqrt{5}$	9	∞
$x+\sqrt{5}$	$-$	0	$+$	$+$	$+$
$x-\sqrt{5}$	$-$	$-$	0	$+$	$+$
$x-9$	$-$	$-$	$-$	0	$+$
Problem	$-$	$+$	$-$	$+$

$\qquad -\sqrt{5} < x < \sqrt{5} \qquad x > 9$

Graphing

$-\sqrt{5} \qquad \sqrt{5} \qquad 9$

I.N. $\quad (-\sqrt{5}, \sqrt{5}) \cup (9, \infty)$ S.B.N. $\{x \mid -\sqrt{5} < x < \sqrt{5} \text{ OR } x > 9\}$

Solve $\dfrac{2}{x-4} \le \dfrac{1}{x+4}$

RHS = 0

$\dfrac{2}{x-4} - \dfrac{1}{x+4} \le 0$

Express LHS as a Single Fraction

$\dfrac{2\,(x+4)}{(x-4)(x+4)} - \dfrac{1\,(x-4)}{(x+4)(x-4)} \le 0$

$\dfrac{2(x+4) - 1(x-4)}{(x+4)(x-4)} \le 0$

$\dfrac{2x+8 - x+4}{(x+4)(x-4)} \le 0$

$x+12=0 \rightarrow x=-12$

$x+4=0 \rightarrow x=-4$

$x-4=0 \rightarrow x=4$

$\dfrac{x+12}{(x+4)(x-4)} \le 0$

Sign chart

x	$-\infty$		-12		-4		4		∞
$x+12$		$-$	$\bullet$	$+$		$+$		$+$	
$x+4$		$-$		$-$	0	$+$		$+$	
$x-4$		$-$		$-$		$-$	0	$+$	
Problem		$-$		$+$		$-$		$+$	

Graphing

-12   -4   4

I.N. $\quad (-\infty, -12] \cup (-4, 4)$

S.B.N. $\quad \{ x \mid x \le -12 \text{ OR } -4 < x < 4 \}$

T

Room temp. varies directly as Square root of the # of people inside. $\quad T = K \cdot \sqrt{N}$

N

Room temp. is $60°$ when there are 9 people in the room.

$60° = K \cdot \sqrt{9}$

$60 = K \cdot 3$

$\rightarrow K = 20$

$\boxed{T = 20\sqrt{N}}$

Find room temp. when there are 25 people in the room.

$T = 20\sqrt{25}$

$= 20 \cdot 5 = 100$

$100°$

The [heat] from a fireplace varies inversely as square of the [distance] from the fire place. $H = \dfrac{K}{D^2}$

Heat is $20°$ when we are are 5 feet from the fireplace. $20 = \dfrac{K}{5^2}$ $20 = \dfrac{K}{25}$ [K=500]

$K = 20(25)$

Find the heat when we are 2 feet from the fire place.

$H = \dfrac{500}{D^2}$

$H = \dfrac{500}{2^2} = \dfrac{500}{4}$

[H = 125]

$125°$

Z varies directly as square root of the sum of x^2 and y^2. $Z = K \cdot \sqrt{x^2 + y^2}$

$Z = 20$ when $x = 3$ & $y = 4$.

$20 = K \cdot \sqrt{3^2 + 4^2}$

$20 = K \cdot \sqrt{25}$

$20 = K \cdot 5$

[K = 4]

find Z when $x = 6$ & $y = 8$.

$Z = 4\sqrt{x^2 + y^2}$

$= 4\sqrt{6^2 + 8^2} = 4\sqrt{36 + 64} = 4\sqrt{100} = 4 \cdot 10 = 40$

Class QZ 5

Factor completely:

1) $x^2 - x - 30 = \boxed{(x-6)(x+5)}$

2) $x^2 - 36 = x^2 - 6^2 = \boxed{(x-6)(x+6)}$

3) $x^3 + 8 = x^3 + 2^3 = \boxed{(x+2)(x^2 - 2x + 4)}$